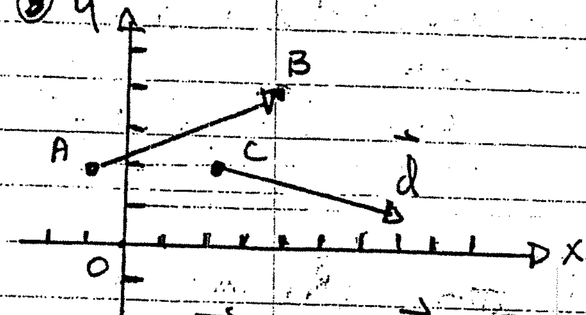
Remarks Vectors having the same orientation are parallel.

Vector properties are invariant, no matter the position of the vector in space. For example, they may be expressed i... of unit vectors and added.

Example ⑧



add the vectors $\vec{AB}$ and $\vec{cd}$. Find $|\vec{R}|$ and θ.

$\vec{R} = \vec{AB} + \vec{cd}$ Note: $\vec{AB}$ and $\vec{cd}$ may be added using head-t... method or analytically. I will add them analytically using unit vector notation. ~~Note~~ ordered pair notation m... be used.

i) $\vec{AB} = \vec{B} - \vec{A}$

$= \vec{OB} - \vec{OA}$

$\vec{OB} = 4\hat{i} + 4\hat{j}$; $\vec{OA} = -\hat{i} + 2\hat{j}$

$\vec{OB} - \vec{OA} = (4\hat{i} + 4\hat{j}) - (-\hat{i} + 2\hat{j}) = 5\hat{i} + 2\hat{j}$

thus $\vec{AB} = 5\hat{i} + 2\hat{j}$ (Note this result may be obtained directly by tra... origin to point 'A')

similarly, $\vec{cd} = \vec{d} - \vec{c}$

$= \vec{od} - \vec{oc}$

$\vec{od} = 7\hat{i} + \hat{j}$

$\vec{oc} = 2\hat{i} + 2\hat{j}$

$\vec{od} - \vec{oc} = (7\hat{i} + \hat{j}) - (2\hat{i} + 2\hat{j}) = 5\hat{i} - \hat{j}$

thus $\vec{cd} = 5\hat{i} - \hat{j}$

then, $\vec{R} = \vec{AB} + \vec{cd} = (5\hat{i} + 2\hat{j}) + (5\hat{i} - \hat{j})$

$= 5\hat{i} + 5\hat{i} + 2\hat{j} - \hat{j}$

$= 10\hat{i} + \hat{j}$

$R = |\vec{R}| = \sqrt{10^2 + 1^2} = \sqrt{101} \approx 10.05$

$\tan\theta = \frac{1}{10}$, $\theta = \tan^{-1}(\frac{1}{10}) \approx 5.71°$

Exercises

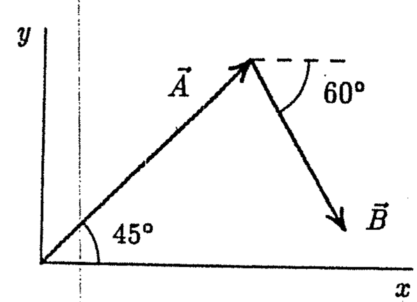
1. Write the magnitude and direction of each of the vectors; sketch
   a.) $\hat{i} + \hat{j}$ b.) $\vec{c} = -2\hat{i} + 3\hat{j}$

2. Find $\vec{R}$ (add vectors), $|\vec{R}|$ and θ

$\vec{A} = 16\hat{i} - 5\hat{j}$
$\vec{B} = 4\hat{i} + 20\hat{j}$
$\vec{C} = 5\hat{i} - 2\hat{j}$

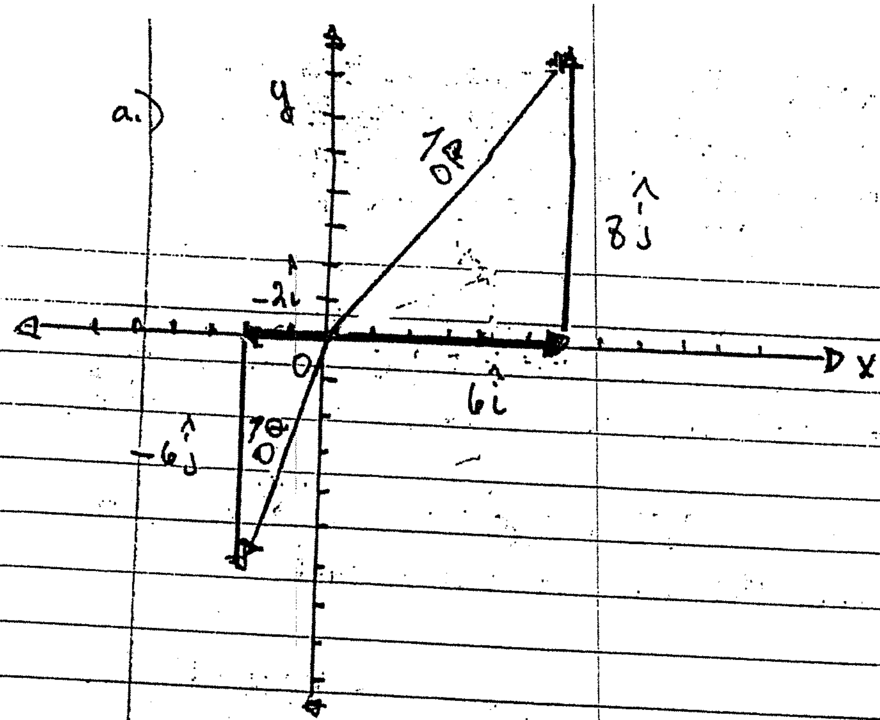
3. Draw a rectangular coordinate system to scale;
   let $\vec{OX} = 7\hat{i} - 4\hat{j}$, $\vec{OY} = 2\hat{i} + 8\hat{j}$
   Add $\vec{OX}$ and $\vec{OY}$ using the head-to-tail method
   measure the length of the Resultant vector with a ruler and the angle with a protractor. Compare your measurements to analytical predictions. Find % error.

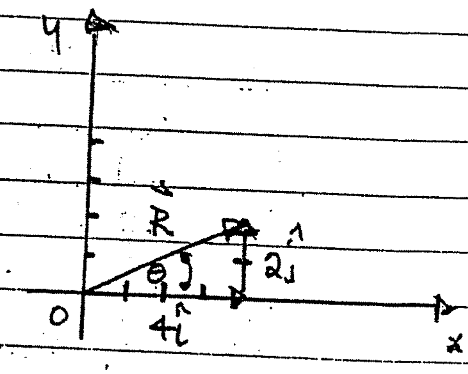
In the diagram, $\vec{A}$ has magnitude 12 m and $\vec{B}$ has magnitude 8 m. The x component of $\vec{A} + \vec{B}$ is about:



A. 5.5 m
B. 7.6 m
C. 12 m
D. 14 m
E. 15 m

a.)





b.) $\vec{R} = \vec{OP} + \vec{OQ}$

$= (6\hat{i} + 8\hat{j}) + (-2\hat{i} - 6\hat{j})$

group together like terms

$= 6\hat{i} - 2\hat{i} + 8\hat{j} - 6\hat{j}$

$\vec{R} = 4\hat{i} + 2\hat{j}$

c.)

$R = |\vec{R}| = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \simeq 4.5$

$\tan\theta = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$

d.) $\vec{R} = (4, 2)$

Remarks ① $\vec{R}$ was found analytically using $\vec{OP}$ and $\vec{OQ}$. You may also use head-to-tail method to find $\vec{R}$.

② $\vec{R}$ may be found analytically using ordered pair notation. Here, $\vec{R} = (6, 8) + (-2, -6) = (4, 2)$

Remarks ①. Vectors may be multiplied by scalars.

Example Let $\vec{A} = 10\hat{i} - 14\hat{j}$

then $3 \cdot \vec{A} = 3(10\hat{i} - 14\hat{j}) = 30\hat{i} - 42\hat{j}$

② Vectors are equal if they have the same magnitude and direction.

REMARKS (continued) Two or more vectors are combined (added) to yield a resultant vector. In our example (4), the unit vectors $5\hat{i}$ and $3\hat{j}$ were added to yield the resultant vector, $\vec{B}$.

REMARKS - A vector may be expressed as the sum of its component vectors. Because $\vec{B} = 2.5\hat{i} + 1.5\hat{j}$, the vectors $2.5\hat{i}$ and $1.5\hat{j}$ are the component vectors of $\vec{B}$.

REMARKS The vectors $2.5\hat{i}$ and $1.5\hat{j}$ were combined to form the vector $\vec{B}$ using the head-to-tail method.

REMARKS Because the x and y axes are mutually perpendicular, the vectors $2.5\hat{i}$ and $.5\hat{j}$ are also mutually perpendicular. Hence, when a vector is expressed in terms of its component unit vectors, a right triangle is formed. Consequently, the properties of a right triangle are used, when appropriate, to make calculations.

Example (5) a.) Find the magnitude of $\vec{B}$, $|\vec{B}|$.

$|\vec{B}| = B = \sqrt{2.5^2 + 1.5^2} = \sqrt{6.25 + 2.25} = \sqrt{8.5} \simeq 2.9$. b.) Find orientation.

$\tan\theta = \frac{1.5}{2.5} \Rightarrow \theta = \tan^{-1}\left(\frac{1.5}{2.5}\right) \approx 31^\circ$

Example (6) $\vec{B}$ may also be represented using ordered pair notation. $\vec{B} = (2.5, 1.5)$ -- this is read as $\vec{B} = 2.5\hat{i} + 1.5\hat{j}$. (Note that the component along the x-axis is written as the 1st term in the ordered pair, and 1.5, the 2nd term, is the component along the y-axis).

Example (7) Let $\vec{OP} = 6\hat{i} + 8\hat{j}$, $\vec{OQ} = -2\hat{i} - 6\hat{j}$.

a.) sketch $\vec{OP}$ and $\vec{OQ}$ (use head-to-tail method to express $\vec{OP}$ and $\vec{OQ}$ as a combination of their component vectors)

b.) Find (and sketch) the resultant vector, R

c.) Find $|\vec{R}|$ and its orientation.

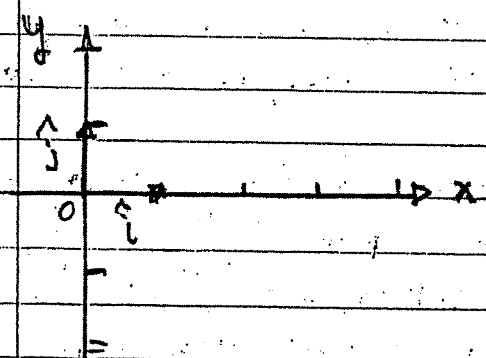
d.) Write $\vec{R}$ using ordered pair notation

OVER →

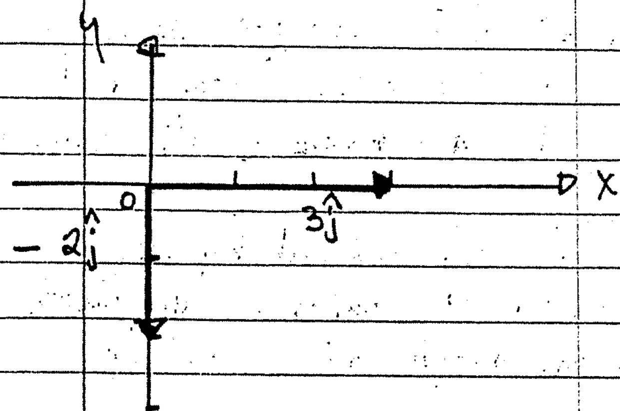
Recall that $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors because their magnitude (siz is 1. The magnitude of a vector is represented using the 'absolute value' symbol - thus $|\hat{i}| = 1$, $|\hat{j}| = 1$, $|\hat{k}| = 1$.

The vectors $\hat{i}$ and $\hat{j}$ are unit vectors along the x and y axes.

($\hat{k}$ is a unit vector along z-axis in a 3-dimensional coordinate system)



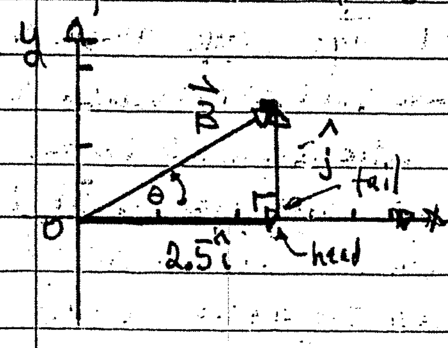
Example (3) Represent the vectors $3\hat{i}$ and $-2\hat{j}$ on the axis below.



Example (4) Find the magnitudes of the vectors $3\hat{i}$ and $-2\hat{j}$.

$|3\hat{i}| = 3$, $|-2\hat{j}| = 2$

Vectors such as $\vec{B}$ in example (1) may be represented as a sum of unit vectors. Thus $\vec{B} = 2.5\hat{i} + 1.5\hat{j}$.



* Remarks When depicting the addition of vectors, note that the tail of one vector is placed at the head of the other vector. (In this example, the tail of $1.5\hat{j}$ is drawn at the head of $2.5\hat{i}$. The segment connecting the tail of $2.5\hat{i}$ to the head of $1.5\hat{j}$ is the vector $\vec{B}$. Note that the tail of $\vec{B}$ is placed at the tail of $2.5\hat{i}$. And the head of $\vec{B}$ is placed at th

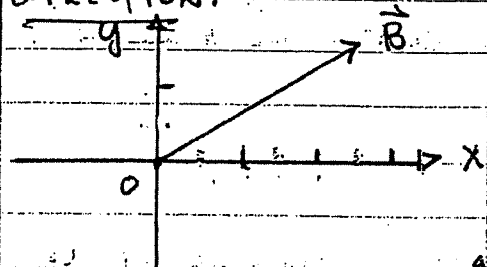
Geometry

Vectors

Vectors are quantities having magnitude and direc
Examples of vectors include force (size and direction)
and velocity (size & direction). Scalars have magnitude on

Vectors may be represented alphanumerically by an arrow drawn over a letter from the alphabet, e.g.; $\vec{a}$, $3\vec{B}$, $\frac{\vec{X}}{2}$ or by a cap drawn over a letter, e.g., $\hat{i}$, $\hat{j}$, $\hat{k}$.
(In the case where a cap is drawn over a letter, the vector is called a unit vector because its magnitude (size) is "1".) A vector may also be represented by a directed line segment whose length represents the magnitude (assume a suitable scale factor), and direction/orientation by an angle (with respect to a coordinate system).

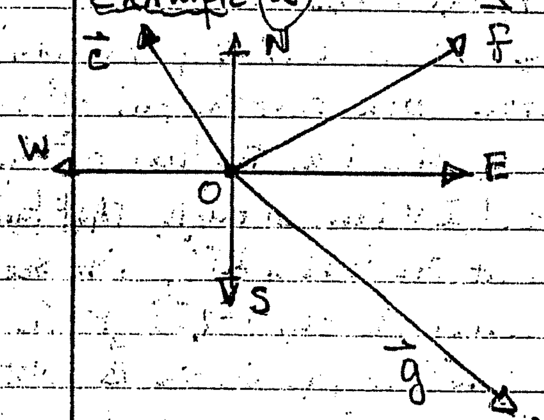
Example ①, the vector $\vec{B}$ is represented by the directed line segment shown. Assuming a scale factor of $15 \frac{\text{Newtons}}{\text{cm}}$, find the magnitude of $\vec{B}$ and its direction.



Note: the tail of $\vec{B}$ is at 'o', the 'head' of $\vec{B}$ is the tip of the arrow.

a.) (hint, use a protractor).

Example ②



Find the orientation of vectors $\overrightarrow{oe}$, $\overrightarrow{of}$ and $\overrightarrow{og}$ (note that when 2 letters are used to represent a vector, the first letter represents the tail and the 2nd letter represents the head).

b.) is it fair to assume that $\overrightarrow{of}$, $\overrightarrow{oe}$ and $\overrightarrow{og}$ have the same magnitude? Why? why not?